

**Conversion Equations Between Fork  
Length and Total Length for Chinook  
Salmon (*Oncorhynchus tshawytscha*)**

by

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## ABSTRACT

Length analyses of Pacific salmon (*Oncorhynchus spp.*) on the west coast of North America typically rely on fork length (FL) measurements. Conversely, size limits for many salmon fisheries are specified in terms of total length (TL). The use of total length in fishery regulations has two ramifications: (1) results from analyses of sample data must be converted to total length prior to implementation as regulations; and (2) fishery size limits must be converted to fork length prior to analysis by models which predict the effect of proposed management regimes upon fish stocks. Accurate formulas for converting from FL to TL, and from TL to FL, are required by fishery managers.

This study uses fork length and total length data for chinook salmon (*O. tshawytscha*) collected by the Washington Department of Fish and Wildlife and the Northwest Indian Fisheries Commission to estimate the relationships between these two measures of length. Three models were considered for estimating the conversion formulas: simple linear regression, geometric mean regression, and an errors-in-variables model. The geometric mean regression model is the method of choice in this study because the results are symmetric and the model accounts for errors in both variables. The geometric mean regression model is shown to be a special case of an errors-in-variables model.

Analysis of covariance found that the relationship between fork length and total length was significantly different among the four data sets analyzed when the entire range of length data was considered ( $35 \text{ cm} \leq \text{FL} \leq 80 \text{ cm}$ ). Our analyses determined that if the data were divided into two length ranges, the hypothesis of equal slopes for the FL:TL relationship among the data sets could not be rejected. The two length ranges were: (1)  $35 \text{ cm} \leq \text{FL} < 68 \text{ cm}$ ; and (2)  $68 \text{ cm} \leq \text{FL} \leq 80 \text{ cm}$ .

The following relationships, based on the geometric mean regression model, for converting from fork length to total length, or total length to fork length, are recommended.

When converting from FL to TL, for lengths in the range of  $35 \text{ cm} \leq \text{FL} < 68 \text{ cm}$ :

$$TL = 1.023 + (1.045 FL)$$

and for lengths in the range of  $68 \text{ cm} \leq \text{FL} \leq 80 \text{ cm}$ :

$$TL = 1.488 + (1.032 FL).$$

When converting from TL to FL, for lengths in the range of  $37 \text{ cm} \leq \text{TL} < 72 \text{ cm}$ :

$$FL = (0.957 TL) - 0.979$$

and for lengths in the range of  $72 \text{ cm} \leq \text{TL} \leq 84 \text{ cm}$ :

$$FL = (0.969 TL) - 1.442 .$$

These conversions necessitate changes in the parameters used in fishery regulation assessment models. Also, the effectiveness of the current size limits in some fisheries may need to be reassessed if the size limit was based upon previous fork length to total length conversion models.

## TABLE OF CONTENTS

|  | <u>PAGE</u> |
|--|-------------|
| ABSTRACT .....                                   | i           |
| LIST OF TABLES .....                             | iv          |
| LIST OF FIGURES .....                            | v           |
| INTRODUCTION .....                               | 1           |
| METHODS .....                                    | 2           |
| Initial Data Analysis .....                      | 2           |
| Comparison of Data Sets .....                    | 3           |
| Models .....                                     | 4           |
| Simple Linear Regression Model .....             | 5           |
| Geometric Mean Regression Model .....            | 6           |
| Errors-in-Variables Model .....                  | 7           |
| RESULTS .....                                    | 9           |
| Initial Data Analysis .....                      | 9           |
| Comparison of Data Sets .....                    | 9           |
| Model Selection .....                            | 17          |
| Geometric Mean Regression Model Parameters ..... | 17          |
| DISCUSSION .....                                 | 26          |
| ACKNOWLEDGMENTS .....                            | 27          |
| REFERENCES CITED .....                           | 28          |
| APPENDIX TABLE 1 .....                           | 30          |

## LIST OF TABLES

| <u>TABLE</u>              |   | <u>PAGE</u>     |
|---------------------------|---|-----------------|
| 1.                        | Sample size (n), mean ( $\bar{x}$ ), standard deviation (s), coefficient of variation (CV), range, and selected correlation coefficients ( $r$ ) for the fork length, total length, and difference between the two length measurements (DIFF) for each of the four data sets examined.....                                    | 10              |
| 2.                        | Mean and standard deviation of total length (TL) for specified intervals of the fork length (FL) data, all data sets combined .....   | 10              |
| 3.                        | Sample size (n), mean ( $\bar{x}$ ), standard deviation (s), coefficient of variation (CV), and selected correlation coefficients ( $r$ ) for the fork length, total length, and difference between the two length measurements (DIFF) for each of the four data sets examined within each of the length ranges examined..... | 18              |
| 4.                        | GMR model parameter estimates and 95% confidence interval for $\hat{b}_{GMR}$ for each length range and data set analyzed.....  | 23              |
| 5.                        | Comparison of predicted total lengths from the GMR FL-to-TL equations for length ranges I ( $35 \text{ cm} \leq \text{FL} < 68 \text{ cm}$ ) and II ( $68 \text{ cm} \leq \text{FL} \leq 80 \text{ cm}$ ).....  | 24              |
| <br><u>APPENDIX TABLE</u> |   | <br><u>PAGE</u> |
| 1.                        | Comparison of fork length (FL) to total length (TL) conversions between the GMR model and the PMFC model .....  | 30              |

## LIST OF FIGURES

| <u>FIGURE</u>  | <u>PAGE</u> |
|--|-------------|
| 1. Comparison of fork length frequencies for each of the four data sets analyzed..   | 11          |
| 2. Comparison of the frequencies of the differences between the total length and the fork length for data pairs in each data set analyzed.....   | 12          |
| 3. Scatter plot of fork length versus total length for the four data sets examined. Data restricted to the range: $35 \text{ cm} \leq \text{FL} \leq 80 \text{ cm}$ .....  | 13          |
| 4. Linear regression lines describing the FL:TL relationship for each of the four data sets examined. Data restricted to the range: $35 \text{ cm} \leq \text{FL} \leq 80 \text{ cm}$ .....  | 15          |
| 5. Plot of the mean difference between TL and FL (DIFF) at each one cm interval of FL. Data restricted to the range: $35 \text{ cm} \leq \text{FL} \leq 80 \text{ cm}$ . Dotted line illustrates the general form of the trend in means as interpreted by the authors.....   | 16          |
| 6. Frequency histogram for FL data used to estimate the relationship between FL and TL. Distributions represent the percent of samples in each length range: $35 \text{ cm} \leq \text{FL} < 68 \text{ cm}$ and $68 \text{ cm} \leq \text{FL} \leq 80 \text{ cm}$ . Contribution of each data set shown, also..... | 19          |
| 7. Estimated slope and 95% confidence interval for the GMR model for each data set. Length range: $35 \text{ cm} \leq \text{FL} \leq 80 \text{ cm}$ .....  | 21          |
| 8. Estimated slope and 95% confidence interval for the GMR model for each data set within the two length ranges .....  | 22          |
| 9. Comparison of GMR fork length to total length conversion lines to the line from the PMFC model.....   | 25          |

## INTRODUCTION

Minimum and maximum size limits are frequently used in fisheries management to regulate the catch. Selection of a size limit may be based upon length-age relationships, length-weight relationships, length histograms, or other analyses of length data. The fork length (FL) is the preferred measurement standard for Pacific salmon (*Oncorhynchus spp.*) due to the ease with which it may be measured (PSC 1989) and its reduced sensitivity, relative to the total length, to the effects of caudal fin fraying and erosion (Bagenal 1978).

Salmon fisheries under the jurisdiction of the Pacific Fishery Management Council (PFMC), and state and tribal salmon fisheries in Washington, Oregon, and California have size limits specified in total length (TL). Because total lengths are used in fishery regulations, the results from analyses of sample data must often be converted to total length prior to implementation. Also, fishery size limits in total length must be converted to fork length prior to analysis by models that predict the effects of proposed management regimes upon fish stocks.

Formulas which accurately convert fork length to total length (and total length to fork length) are necessary for accurate modeling of fisheries with length-based retention regulations. Fishery managers in Washington State have not agreed on a length conversion formula for chinook salmon (*O. tshawytscha*). One of the current conversion formulas used is based on a simple linear regression and data collected from the Washington coastal troll fishery in 1953 [Appendix 1 in Reed (1972)<sup>1</sup>]. This conversion formula is thought to overestimate total length because the total length was collected by forcing the caudal peduncle down so that the tip of the caudal fin was aligned with a meter stick (Dr. S. Moore, Washington Department of Fish and Wildlife, personal communication). More fork length and total length data have been collected during the past decade. For these data, samplers consistently measured total length with the tail in its natural position. This paper analyzes these new data and evaluates three different linear statistical models for their effectiveness in describing the relationship between fork length and total length (FL:TL relationship).

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<sup>1</sup> This work was reported by the Pacific Marine Fisheries Commission (PMFC) and is subsequently referred to as the PMFC model.

## METHODS

The data consisted of pairs of length measurements from individual fish. Each measurement pair consisted of a fork length and a total length measurement. Four data sets were analyzed. Descriptions of each data set and the abbreviation used to refer to each set subsequently in the report follow:

- Set 1. Data collected by the Washington Department of Fish and Wildlife (WDFW) in January through December of 1987, consisting of 3,273 length measurement pairs from chinook salmon collected during surveys of recreational hook-and-line fisheries in Puget Sound (abbreviated as WDFW87);
- Set 2. Data collected by the Northwest Indian Fisheries Commission (NWIFC) in March 1987, consisting of 98 length measurement pairs from chinook salmon harvested with troll gear by treaty Indians at Port Angeles and Neah Bay (abbreviated as NWIFC87);
- Set 3. Data collected by WDFW in September and October of 1994, consisting of 203 length measurement pairs from chinook salmon landed during purse seine test fisheries for coho salmon (*O. kisutch*) at Apple Cove in Puget Sound (abbreviated as WDFW94); and
- Set 4. Data collected by WDFW in September and October of 1995, consisting of 378 length measurement pairs from chinook salmon landed during purse seine test fisheries for coho salmon at Apple Cove and Edward's Point in Puget Sound (abbreviated as WDFW95).

### Initial Data Analysis

All measurements made by the WDFW were in centimeters (cm) and recorded to the nearest cm. The measurements made by the NWIFC were in inches and recorded to the nearest 1/8": these measurements were converted to cm prior to analysis. Before analysis, each data set was screened for possible recording errors. All data pairs where the fork length was greater than the total length were considered recording errors and removed from the data. The difference between total length and fork length (TL - FL) was calculated for each data pair. Differences between total length and fork length greater than 10 cm (TL - FL > 10 cm) were extremely rare in the data (15 pairs out of 3,952 total pairs). These pairs were considered outliers and were removed from the data, also.

After we had screened each data set, the mean, standard deviation, coefficient of variation, and range (minimum value to maximum value) for each set were calculated for the variables of interest: fork length, total length, and difference between the two (TL - FL = DIFF). In addition, the parametric Pearson's correlation coefficient (*r*) was calculated



between FL and TL and between FL and DIFF. Length frequency histograms were used to compare the fork length data from each data set. Frequency histograms were also used to compare the distribution of values for DIFF from each data set.

### Comparison of Data Sets

Since the biological relationship between fork length and total length is not well understood, we limited the range of lengths analyzed to those of interest for fishery management. This prevents lengths outside the range of interest from influencing the functional relationship estimated between fork length and total length and should allow for more accurate estimates of the FL:TL relationship for lengths within the range. Current length limits range from about 35 cm to 80 cm. Therefore, all data pairs with a fork length less than 35 cm or greater than 80 cm were removed from each data set. Scatter plots were used to examine the bivariate relationship between the different variables in the reduced data sets.

A determination of the error structure of the data was required to select the proper model for analysis. If the variance structure is homoscedastic (the variance of the  $Y$  data is constant over the range of the  $X$  data) the error structure is normal. If the variance structure is heteroscedastic (the variance of the  $Y$  data is not constant over the range of the  $X$  data) the error structure is usually log-normal. We divided the FL data in the range from 35 cm to 80 cm into one 5-cm and four 10-cm intervals. The mean and standard deviation of the corresponding total lengths within each of the length intervals was calculated to examine the variance structure of the FL:TL relationship. Box's test and Levene's test for the homogeneity of variance (Milliken and Johnson 1992) were conducted on the total length data within the fork length categories. Both of these tests are robust to departures from normality (Milliken and Johnson 1992).

Our first step in examining the FL:TL relationship was to determine if it was appropriate to combine the four data sets. If the relationship between FL and TL is different among the four data sets it is not appropriate to combine them to estimate an overall FL:TL relationship. We used a simple linear model and analysis of covariance (ANCOVA) to determine if the FL:TL relationships were similar among the four data sets. The assumptions required for the simple linear regression model are described in the next section of this report. Use of the linear regression model allowed us to use analysis of covariance to simultaneously compare the FL:TL relationships among the data sets (Milliken and Johnson 1996). We followed procedures outlined by Milliken and Johnson (1996) to conduct the ANCOVA. We are not aware of similar procedures that could be used with the other models examined in this report. Because the linear model fit the data well, we do not expect that its use introduced any errors into our analysis or biased our conclusions.

## Models

Three models were considered for estimating the FL-to-TL and TL-to-FL conversion equations:

- simple linear regression (SLR) model;
- geometric mean regression (GMR) model; and
- errors-in-variables (EIV) model.

Schnute (1984) identified three fundamental properties that should be satisfied by a model expressing the relationship between bivariate data: They are:

- the results should be symmetric in  $X$  and  $Y$ ;
- the results should be scale independent; and
- the results should be robust to clusters of samples toward either end of the distribution.

A regression-type model is symmetric in  $X$  and  $Y$  if the slope estimate for the regression of  $Y$  on  $X$  is the reciprocal of the slope estimate for the  $X$ -on- $Y$  regression. For example, a FL can be converted to TL using the results of a FL-on-TL regression. This predicted TL can then be converted back to a FL using the results of the TL-on-FL regression. In a symmetric model, this predicted FL will be the FL that was initially used in the FL-on-TL regression. If the model is not symmetric, the predicted FL will be different from the original FL.

A model is scale independent when the slope estimated from the  $X$ - $Y$  data is not influenced by the measurement units of the data. For example, if a model is scale independent then a regression calculated from FL:TL data measured in centimeters will result in the same estimated slope if the data are converted to inches before analysis.

Frequently when naturally occurring populations are randomly sampled there is a concentration of observations near one end of the frequency distribution of the observations and progressively fewer observations toward the other end, or there may be several modes in the frequency distribution scattered throughout the range of data (Ricker 1973). A good  $X$ -on- $Y$  regression model should be robust (not heavily influenced) to these concentrations of observations.

The major assumptions required for each model and a discussion of the suitability of each model to the FL-to-TL conversion problem follow.

### Simple Linear Regression Model:

The simple linear regression model is the most widely used method because of ease of use and familiarity to most people. The parameters of the simple linear regression model are usually estimated by the method of least squares (Draper and Smith 1981). Because the methods used to estimate the parameters of a SLR are described in detail in most introductory statistical texts they are not presented here. The four major assumptions required for a SLR model are (Draper and Smith 1981):

1. The relationship between the  $X$  (independent) and  $Y$  (dependent) variables can be described by a linear function;
2. For any value of  $X$ , the corresponding  $Y$  values are independently and normally distributed and this distribution has been sampled at random;
3. The variance of  $Y$  is the same for any value of  $X$ , i.e., the variances are homoscedastic; and
4. The independent  $X$  variable is measured without error relative to the dependent  $Y$  variable.

There are two primary criticisms of simple linear regression as a model for length measurement conversions. The first is the SLR model is not symmetric in  $X$  and  $Y$  (Ricker 1973; Schnute 1984). The SLR model is not symmetric because it assigns a special role to one variate. If a given FL is converted to TL with a FL-on-TL regression and subsequently the TL predicted from this regression is used in a TL-on-FL regression, the new predicted FL will usually not be the one originally input into the FL-on-TL regression. The predicted FL will be larger than the original FL if it was less than the mean FL of the original FL data used to estimate the predictive regression, or it will be less than the original FL if it was greater than the mean FL of the original data (Ricker 1973). Although the SLR model is scale independent (Schnute 1984), it is not robust to clusters of samples toward either end of the distribution (Ricker 1973).

The other major criticism of the SLR model concerns the fourth assumption (above): the independent  $X$  variable is assumed to be measured without error relative to the dependent  $Y$  variable. Obviously in the FL:TL relationship we expect that there is some error in the measurement of both lengths. We have no basis to assume that fork length is measured without error with respect to TL (or vice versa). If measurement error is present in the  $X$  variable (as well as the  $Y$  variable), then least-squares regression estimates of the model parameters (the slope and the intercept) and the estimates of the variance of these parameters will be biased (Fuller 1987).

### Geometric Mean Regression Model:

The geometric mean regression was advocated by Ricker (1973) as a method to incorporate measurement errors in the  $X$  variable into regressions. The GMR model is also referred to as the standard (or reduced) major axis method (Jolicoeur 1975). Ricker (1973) stated that the GMR model is superior to the SLR model when estimating conversion factors between different length measurements because “its estimate has no systematic bias related to the range of lengths represented in the sample”. This is in contrast to a simple linear regression whose slope estimate tends to increase as the range of lengths in the sample increases.

The slope of the GMR regression is estimated by the ratio of the sample standard deviations of the  $X$  and  $Y$  variables (Ricker 1973). Specifically, the slope ( $\beta$ ) is estimated by (Ricker 1973):

$$\hat{b}_{GMR} = (\text{sign } r) \left[ \frac{\text{sample standard deviation for } Y}{\text{sample standard deviation for } X} \right] \quad [1]$$

and the intercept ( $\alpha$ ) by:

$$\hat{a}_{GMR} = \bar{Y} - (\hat{b}_{GMR} \bar{X}) \quad [2]$$

where  $\text{sign } r$  is the sign ( $\pm$ ) of the parametric correlation coefficient between  $X$  and  $Y$  and  $\bar{X}$  and  $\bar{Y}$  are the sample means. Jolicoeur (1990) describes how to construct confidence intervals for  $\hat{b}_{GMR}$ . The major assumptions of the GMR model proposed by Ricker are (Ricker 1973; Schnute 1984):

1. The relationship between the  $X$  and  $Y$  variables can be described by a linear function;
2. The  $X$  and  $Y$  data are mutually independent pairs from a bivariate normal distribution that has been sampled at random; and
3. There is measurement error present in both the  $X$  and  $Y$  variables.

The GMR model has two advantages relative to the SLR model. First, the GMR model is symmetric in  $X$  and  $Y$  (Ricker 1973; Schnute 1984). Secondly, the GMR model is robust, relative to the SLR model, to clusters of observations in the frequency distributions of the data (Ricker 1973). The GMR model is also scale independent (Kimura 1992).

However, the GMR model “has been the center of significant controversy in the literature” (Kimura 1992). [Also see, Jolicoeur (1975), Ricker (1975), Schnute (1984), and Jolicoeur (1990) for additional discussions of this controversy.] One criticism of the GMR model is

that its slope coefficient is inconsistent<sup>2</sup> and this inconsistency can result in bias when measurement error is large and the estimated slope is small (Sprent 1969). Also, Jolicoeur (1975; 1990) demonstrates that the confidence interval for the slope estimated from GMR can be too narrow in many situations, especially when there is not a strong relationship between  $X$  and  $Y$ . Jolicoeur (1990) recommends that the GMR should be restricted to those cases when:

- the analysis is restricted to the original values of the  $X$  and  $Y$  data (the data are not log transformed);
- the  $X$  and  $Y$  data have a bivariate normal distribution;
- sample size is at least 20 cases; and
- $r$  between  $X$  and  $Y$  is at least 0.60.

#### Errors-In-Variables Model:

Kimura (1992) presents a good discussion of the application of errors-in-variables models to allometric problems similar to the FL-to-TL conversion problem. EIV models are a class of models that incorporate error in the measurement of both the  $X$  and  $Y$  data. It should be noted that the GMR model (or standard major axis method) is a special case of the linear functional regression models applicable to errors-in-variables models (Kimura 1992). Models of this type all require an assumption about the measurement variances of the  $X$  and  $Y$  data. Either the measurement variances are assumed to be known or the ratio of the measurement error variances (called  $\lambda$ ) is assumed known. The parameter  $\lambda$  is defined by:

$$\lambda = \frac{S_Y^2}{S_X^2}$$

where  $S_Y^2$  and  $S_X^2$  are the measurement error variances for the  $Y$  and  $X$  data, respectively. In the bivariate situation, knowledge of  $\lambda$  allows an unbiased estimator of  $\beta$  to be constructed and the application of normal theory for hypothesis testing and confidence interval construction (Fuller 1987). In the GMR model,  $\lambda$  is assumed to be approximated by the ratio of the sample variances of the  $Y$  and  $X$  data (Jolicoeur 1990; Kimura 1992). The assumptions for the EIV model are identical to those of the GMR model (remember that the GMR model is a member of the same class of models).

The EIV model we examined is often referred to as the ordinary major axis method (Jolicoeur 1975, 1990; Kimura 1992). For this model,  $\lambda$  is assumed equal to 1.0. We feel that this is a reasonable assumption for the FL:TL data as both lengths were measured in the same units (primarily cm) and the differences between the two measurements were

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<sup>2</sup> A method of estimation is consistent when the estimated value becomes equal to the population value as the sample size approaches the population size.

relatively small (1 to 10 cm) so the measurements are equal in magnitude. Therefore, we expect that the two types of measurements have approximately equal measurement error. Kimura (1992) states that in many cases of allometry it is reasonable to assume  $\lambda = 1.0$ .

Fuller (1987) and Jolicoeur (1990) present the maximum likelihood estimate for the slope of this EIV model ( $\hat{b}_{EIV}$ ):

$$\hat{b}_{EIV} = \frac{(s_Y^2 - |s_X^2|) + [(s_Y^2 - |s_X^2|)^2 + 4|s_{XY}^2|]^{1/2}}{2s_{XY}}$$

where,  $s_Y^2$  = the sample variance for the  $Y$  data;  
 $s_X^2$  = the sample variance for the  $X$  data; and  
 $s_{XY}$  = the sample covariance for the  $X$ - $Y$  data.

Fuller (1987) and Jolicoeur (1990) describe how to construct confidence intervals for  $\hat{b}_{EIV}$ . Jolicoeur (1990) recommends this EIV model (ordinary major axis method) when the  $X$ - $Y$  data appear to follow a bivariate log-normal distribution. This model has been criticized as being scale dependent. However, Kimura (1992) has shown that when  $\lambda$  is properly rescaled before checking the model for scale independence, this form of the EIV model is scale independent.

Schnute (1984) proposed a series of EIV models appropriate for bivariate data that follow a log-normal distribution. These models require no explicit assumption about  $\lambda$ . From our analyses, we concluded that the FL:TL data more closely followed a bivariate normal rather than a bivariate log-normal distribution. Therefore, these models were not investigated further.

## RESULTS

### Initial Data Analysis

The WDFW87 data set had the highest incidence of points considered recording errors (FL recorded as  $\geq$  TL) or as outliers (DIFF  $> 10$  cm). About one percent of the data pairs were removed from the WDFW87 data set: 18 data pairs had FL  $\geq$  TL and 15 data pairs had DIFF  $> 10$  cm. No data pairs were removed from the NWIFC87 and WDFW94 data sets. One data pair was removed from the WDFW95 data set because the FL was  $\geq$  TL.

The mean ( $\bar{x}$ ), standard deviation (s), coefficient of variation (CV), range, and selected correlation coefficients ( $r$ ) for each data set for the three variables of interest are shown in Table 1. The WDFW87 data set has the largest mean values for all three variables. The WDFW95 data set had the largest coefficient of variation for all three variables. The WDFW87 data set had a broad distribution of fork lengths and no pronounced peak to its distribution (Figure 1). The NWIFC87 and WDFW94 data sets had narrower distributions of fork lengths and had clear peaks to their frequency distributions. The WDFW95 data set was skewed to the right with peak frequencies in the smaller fork length ranges but with an extended distribution into the larger fork lengths (Figure 1). The WDFW87 and WDFW94 data sets had similar distributions for DIFF (Figure 2): both data sets had peaks in the  $3 \text{ cm} < \text{DIFF} \leq 4 \text{ cm}$  category. The other two data sets (NWIFC87 and WDFW95) had a peak in the  $2 \text{ cm} < \text{DIFF} \leq 3 \text{ cm}$  category.

### Comparison of Data Sets

There was a strong linear relationship between FL and TL in all four data sets (see  $r$  values in Table 1). For the comparison of data sets, the data were restricted to the range of lengths of interest,  $35 \text{ cm} \leq \text{FL} \leq 80 \text{ cm}$ . The scatter plot of the combined data in this range demonstrates the strong linear relationship between FL and TL (Figure 3). For this restricted range of length data,  $r$  values between FL and TL ranged from 0.990 to 0.997 for the individual data sets and  $r$  for the data combined was 0.996.

The spread of the data in Figure 3 indicates a normal error variance structure for the FL:TL data. The data are tightly clustered in the vertical ( $Y$ -axis) direction throughout the range of the FL data. Although the spread of the TL values along the  $Y$ -axis expands slightly as the values of FL increase, the increase in spread is relatively small. Excluding the smallest length category ( $35 \leq \text{FL} \leq 40 \text{ cm}$ ), the standard deviations of TL in the other four FL categories range from 2.95 to 3.15 (Table 2). When the smallest length category is included in the analysis, both Box's and Levene's tests for the homogeneity of variances are rejected (both  $P < 0.001$ ). When the smallest length category is omitted from the analysis, neither test for the homogeneity of variances is rejected (both  $P > 0.100$ ).

Table 1. Sample size (n), mean ( $\bar{x}$ ), standard deviation (s), coefficient of variation (CV), range, and selected correlation coefficients (r) for the fork length, total length, and difference between the two length measurements (DIFF) for each of the four data sets examined.

| Variable             | Statistic | Data Set |             |         |          |
|----------------------|-----------|----------|-------------|---------|----------|
|                      |           | WDFW87   | NWIFC87     | WDFW94  | WDFW95   |
| Fork Length<br>(FL)  | n         | 3,240    | 98          | 203     | 377      |
|                      | $\bar{x}$ | 71.1     | 67.4        | 49.6    | 48.4     |
|                      | s         | 13.1     | 6.9         | 8.6     | 13.7     |
|                      | CV        | 18.4%    | 10.2%       | 17.3%   | 28.3%    |
|                      | range     | 26 - 120 | 56.5 - 94.1 | 31 - 70 | 21 - 100 |
| Total Length<br>(TL) | $\bar{x}$ | 74.9     | 70.3        | 53.2    | 51.4     |
|                      | s         | 13.3     | 6.8         | 9.1     | 14.1     |
|                      | CV        | 17.7%    | 9.6%        | 17.1%   | 27.4%    |
|                      | range     | 27 - 130 | 59.7 - 97.5 | 34 - 75 | 23 - 103 |
| DIFF                 | $\bar{x}$ | 3.81     | 2.93        | 3.54    | 2.96     |
|                      | s         | 1.11     | 0.74        | 0.89    | 0.95     |
|                      | CV        | 29.0%    | 25.2%       | 25.0%   | 32.0%    |
|                      | range     | 1 - 10   | 0.95 - 5.72 | 1 - 6   | 1 - 6    |
| FL, TL               | r         | 0.997    | 0.994       | 0.997   | 0.998    |
| FL, DIFF             | r         | 0.113    | -0.242      | 0.524   | 0.376    |

Table 2. Mean and standard deviation of total length (TL) for specified intervals of the fork length (FL) data, all data sets combined.

| Fork Length Category (cm)   | Mean TL | Standard Deviation | Sample Size |
|-----------------------------|---------|--------------------|-------------|
| $35 \leq \text{FL} \leq 40$ | 40.1    | 2.02               | 131         |
| $40 < \text{FL} \leq 50$    | 49.6    | 3.15               | 267         |
| $50 < \text{FL} \leq 60$    | 59.5    | 2.95               | 824         |
| $60 < \text{FL} \leq 70$    | 69.5    | 3.12               | 910         |
| $70 < \text{FL} \leq 80$    | 79.3    | 3.09               | 891         |



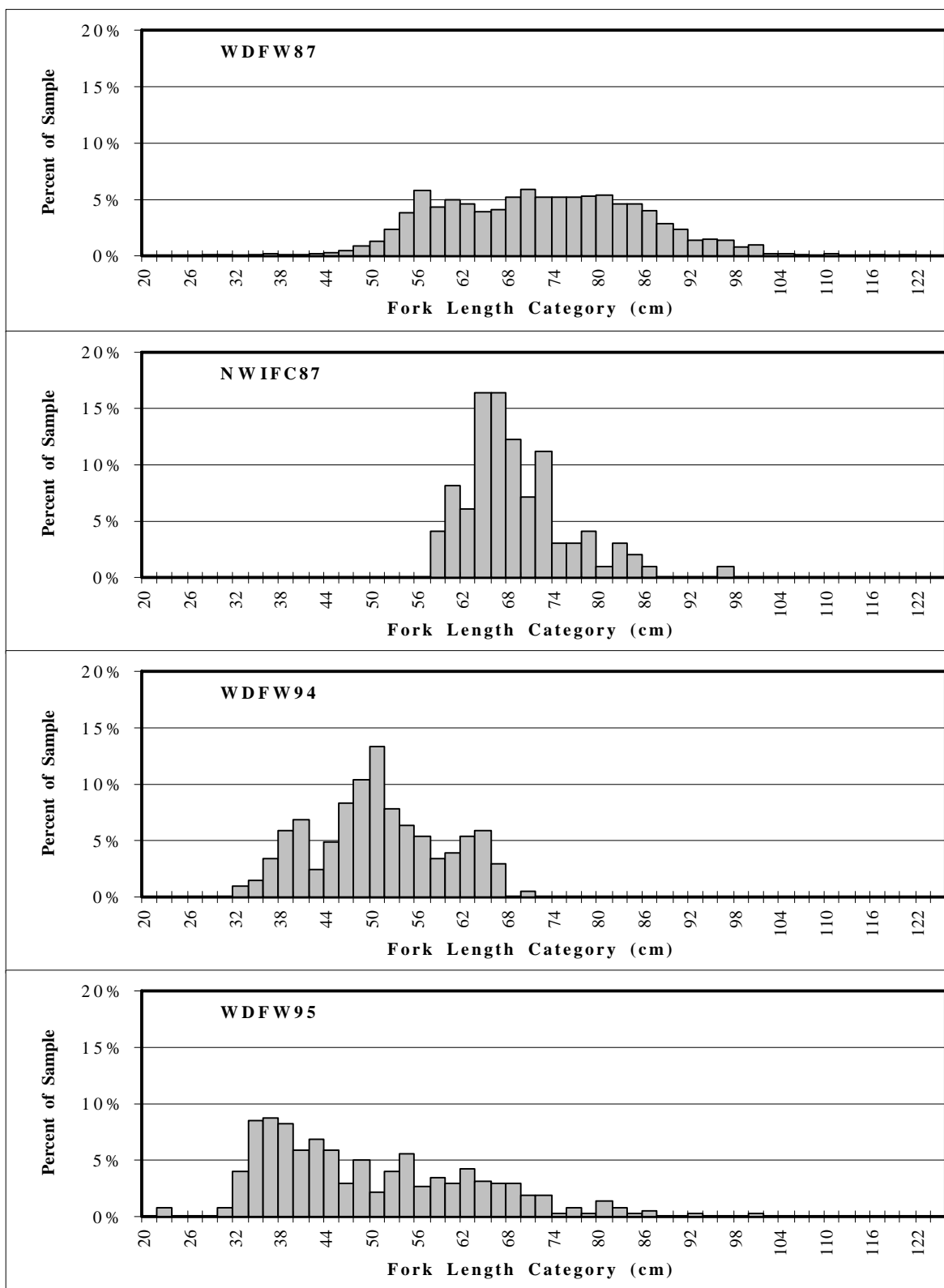


Figure 1. Comparison of fork length frequencies for each of the four data sets analyzed.

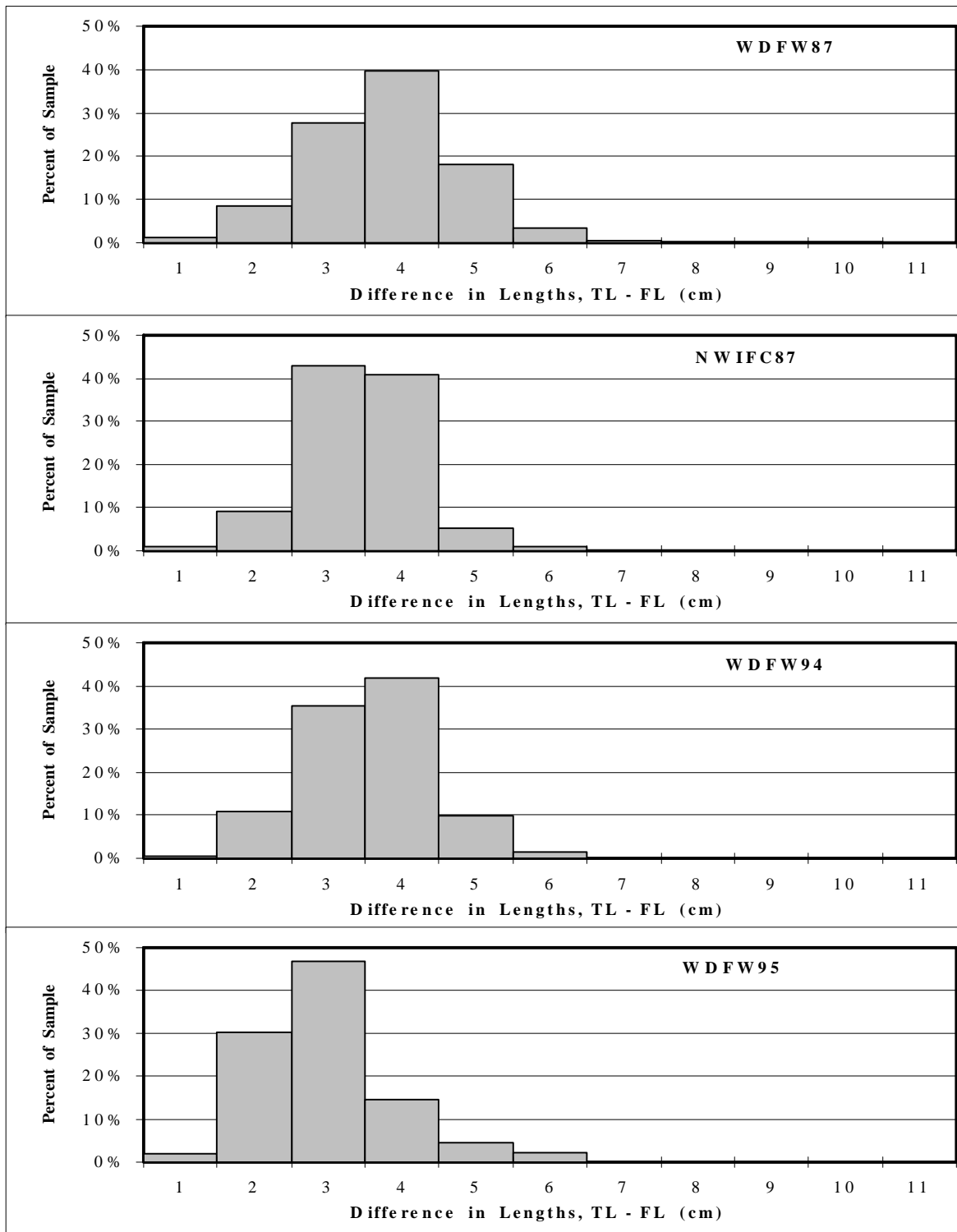


Figure 2. Comparison of the frequencies of the differences between the total length and the fork length for data pairs in each data set analyzed.

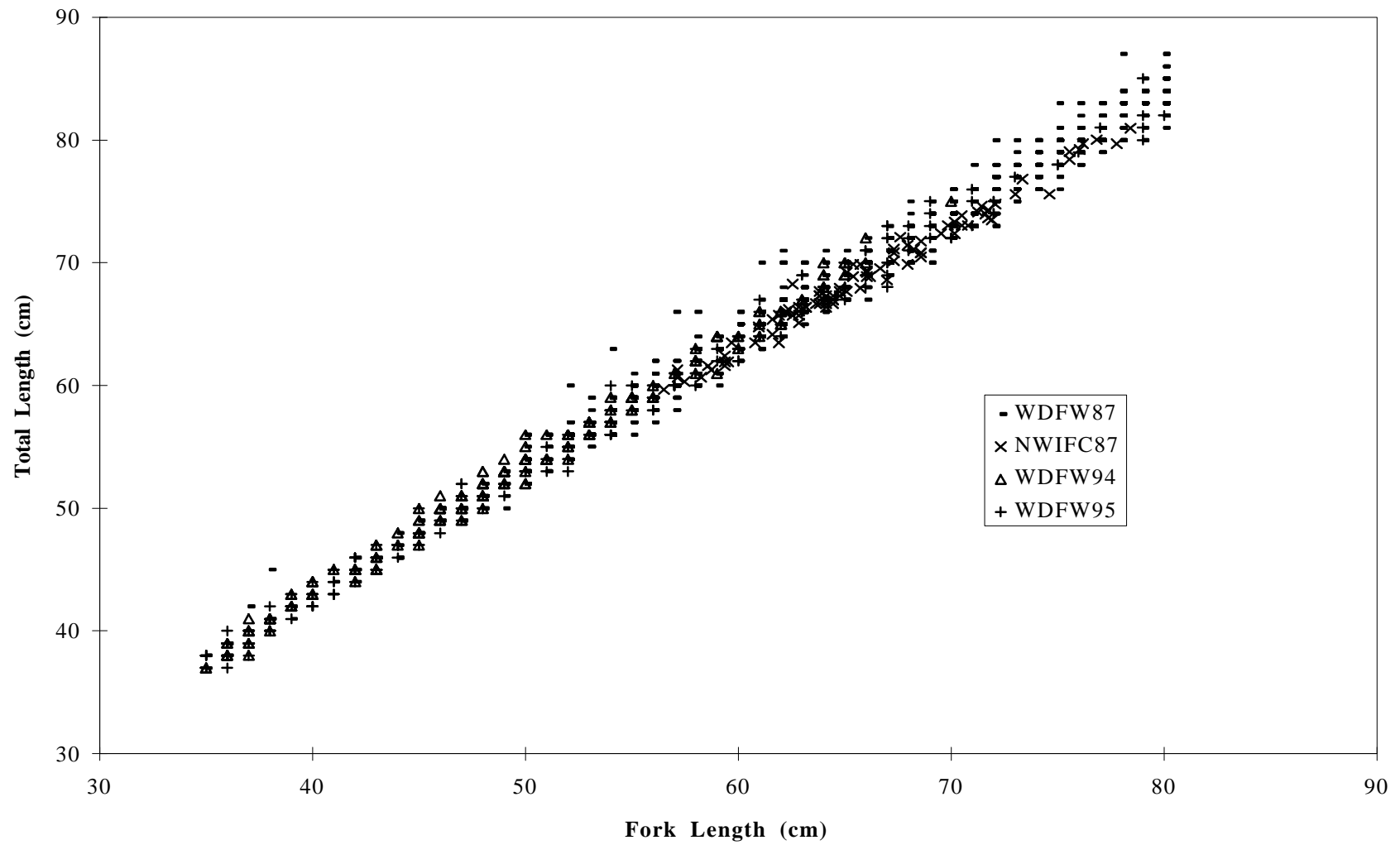


Figure 3. Scatter plot of fork length versus total length for the four data sets examined. Data restricted to the range:  $35 \text{ cm} \leq \text{FL} \leq 80 \text{ cm}$ .

Less than 5% of the data points are in the smallest fork length category, therefore, we feel the influence of these observations on the error variance structure is minimal. Based upon the strong linear correlation between FL and TL, a visual examination of the data, the relatively constant standard deviation of TL throughout the four FL categories  $> 40$  cm, and the results of the homogeneity of variance tests, we concluded that it was reasonable to assume that the FL:TL data have a normally distributed error variance structure.

The assumption of normally distributed error variances allowed us to use analysis of covariance to compare the FL:TL relationships among the four data sets without transforming the data. The ANCOVA of the restricted range data sets ( $35 \text{ cm} \leq \text{FL} \leq 80 \text{ cm}$ ) rejected the hypothesis of equal slopes for the FL:TL relationship among the different data sets ( $P < 0.001$ ). Figure 4 shows the four SLR regression lines estimated from the data sets. The lines are not parallel and all four lines intersect at least one other line within the length range examined. Because of the significant difference in the FL:TL relationship among the data sets it is not appropriate to combine them.

We next examined the relationship between FL and DIFF (the difference between TL and FL). Because nearly all the data were measured to the nearest cm, DIFF took on a limited number of values at each FL value. We calculated the mean DIFF at each one cm interval of FL (the NWIFC87 measurements were rounded to the nearest whole cm) and plotted these (Figure 5). Mean DIFF generally increased as FL increased but appeared to reach a plateau near the end of the length range. The dotted lines in Figure 5 illustrate the general form of the trend in our interpretation. Despite the considerable fluctuation of the mean values around these lines, we believe it is clear that two different relationships exist. Based on this plot, we concluded that the relationship between FL and TL is different for the larger fork lengths compared to the smaller lengths. The differences in the distribution of the larger fork lengths among the data sets (see Figure 1) may be the cause of the differences among the slopes for the FL:TL relationship.

Based on Figure 5, we repeated the ANCOVA with each data set restricted to fork lengths from 35 cm to 60 cm. With these reduced data sets the hypothesis of parallel slopes was not rejected by ANCOVA ( $P = 0.383$ ). Since the selection of 60 cm was a somewhat arbitrary choice, we sequentially increased the maximum length of the FL range by one cm and repeated the ANCOVA. The hypothesis of equal slopes among the data sets was not rejected by the ANCOVA until the range was extended to  $35 \text{ cm} \leq \text{FL} \leq 68 \text{ cm}$ . When the maximum FL in the range was less than 68 cm the hypothesis of equal slopes was not rejected ( $P = 0.142$ ). When the maximum FL in the range was  $\geq 68 \text{ cm}$  the hypothesis of equal slopes was rejected ( $P = 0.028$ ). This indicated that the slopes of the relationship between FL and TL for lengths in the range  $35 \text{ cm} \leq \text{FL} < 68 \text{ cm}$  were equal among the data sets and the data sets could be combined in this length range for analysis.

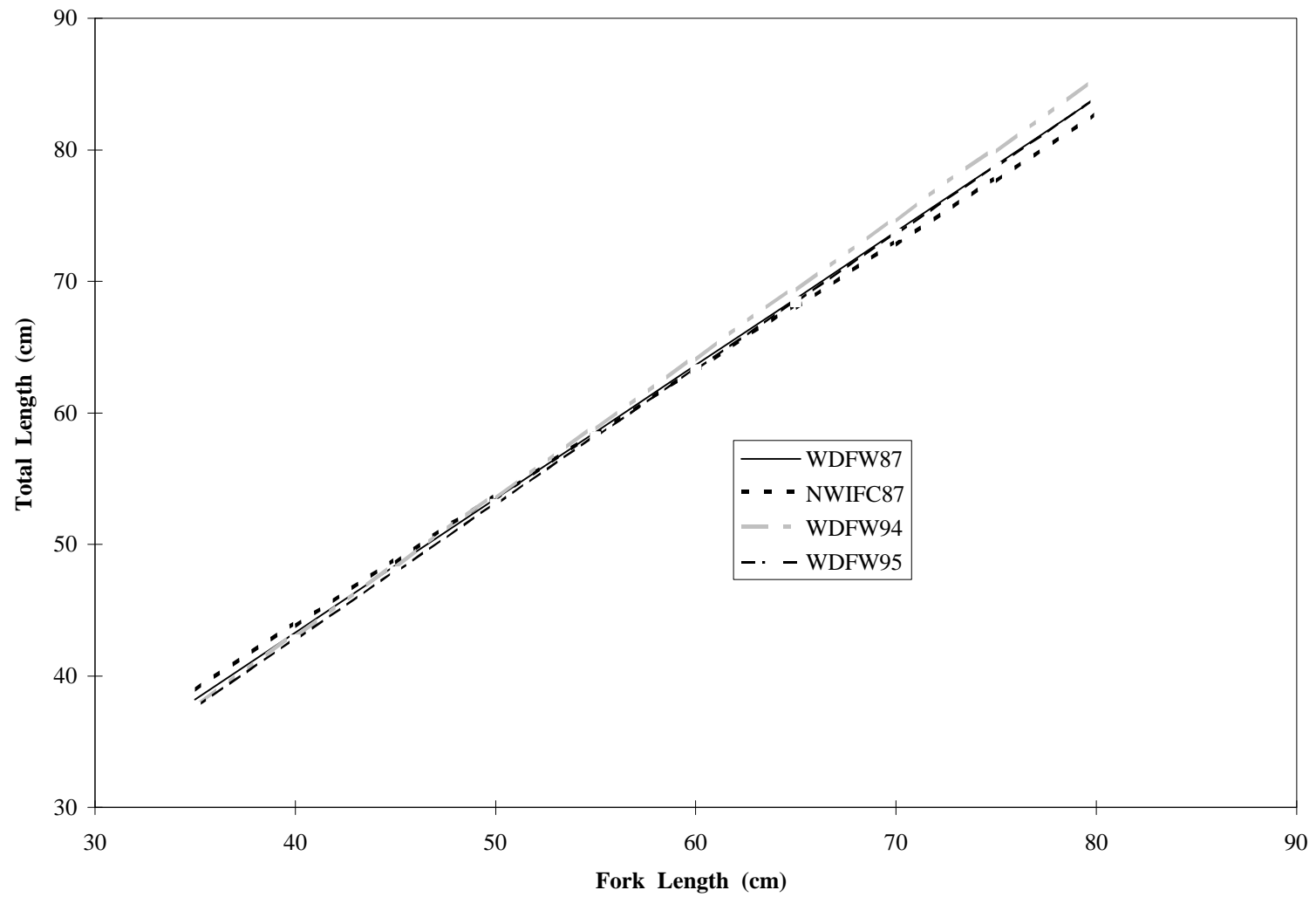


Figure 4. Linear regression lines describing the FL:TL relationship for each of the four data sets examined. Data restricted to the range:  $35 \text{ cm} \leq \text{FL} \leq 80 \text{ cm}$ .

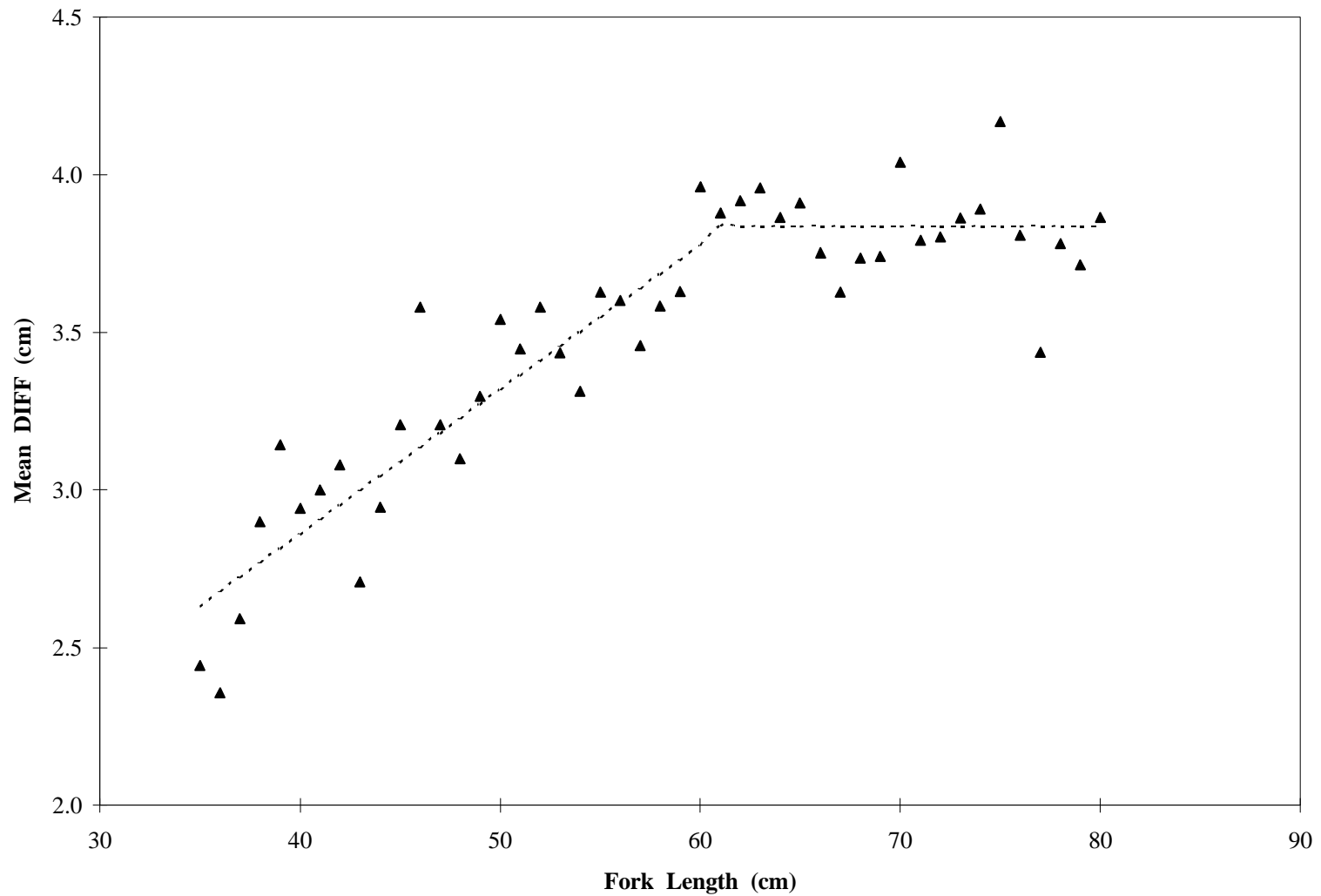


Figure 5. Plot of the mean difference between TL and FL (DIFF) at each one cm interval of FL. Data restricted to the range:  $35 \text{ cm} \leq \text{FL} \leq 80 \text{ cm}$ . Dotted line illustrates the general form of the trend in means as interpreted by the authors.

An ANCOVA of the data above this range ( $68 \text{ cm} \leq \text{FL} \leq 80 \text{ cm}$ ) was not significant either ( $P = 0.381$ ). This indicated the slopes of the relationship between FL and TL for lengths in this range were equal among the data sets and the data sets could be combined in this length range for analysis, also.

In summary, we concluded that the data need to be divided into two length ranges in order to combine the four data sets for analysis. Within each length range, the hypothesis of equal slopes among the four data sets for the FL:TL relationship could not be rejected. The two length ranges were:  $35 \text{ cm} \leq \text{FL} < 68 \text{ cm}$  (length range I) and  $68 \text{ cm} \leq \text{FL} \leq 80 \text{ cm}$  (length range II). The mean, standard deviation, coefficient of variation, and selected correlation coefficients for each data set for these two length ranges are shown in Table 3. Figure 6 shows the distribution of fork lengths and the proportional contribution of each data set to these two ranges of FL data.

### Model Selection

We next decided on the most appropriate model to estimate the FL:TL relationship. We rejected the SLR model because it is not symmetric and does not account for measurement error in the  $Y$  variable. The GMR and EIV models do not have these limitations. Jolicoeur (1990) recommends the EIV model when the data have a bivariate log-normal distribution. Based on our examination of the data, we previously concluded that it was reasonable to assume that the FL:TL data have a normally distributed error variance structure. The FL:TL data correspond well to the criteria outlined by Jolicoeur (1990) for selecting the GMR model: (i) the proposed FL:TL analysis uses the original values of the  $X$  and  $Y$  data (the data are not log transformed); (ii) the  $X$  and  $Y$  data have a bivariate normal distribution; (iii) the sample size exceeds 20 cases (1,857 cases for the analysis of the smaller fork lengths and 1,166 cases for the analysis of the larger fork lengths); and (iv)  $r$  between FL and TL is greater than 0.60 for both FL ranges (0.994 for length range I and 0.961 for length range II). Also, Ricker (1973) recommended the GMR model for estimating conversion factors between different length measurements. Therefore, we selected the GMR model.

### Geometric Mean Regression Model Parameters:

The parameters of the GMR model were calculated using equations 1 and 2 and the 95% confidence interval for  $\hat{b}_{GMR}$  was estimated using the methods of Jolicoeur (1990). Separate models were estimated for the two length ranges determined by the ANCOVA:  $35 \text{ cm} \leq \text{FL} < 68 \text{ cm}$  and  $68 \text{ cm} \leq \text{FL} \leq 80 \text{ cm}$ . Within each of these length ranges, and for the entire restricted range data set ( $35 \text{ cm} \leq \text{FL} \leq 80 \text{ cm}$ ), separate analyses were conducted for each data set to see if the estimates of  $\hat{b}_{GMR}$  supported the conclusions of the ANCOVA.

Table 3. Sample size (n), mean ( $\bar{x}$ ), standard deviation (s), coefficient of variation (CV), and selected correlation coefficients (r) for the fork length, total length, and difference between the two length measurements (DIFF) for each of the four data sets examined within each of the length ranges examined.

| Variable             | Statistic | Data Set: 35 cm ≤ FL < 68 cm |         |        |        |          |
|----------------------|-----------|------------------------------|---------|--------|--------|----------|
|                      |           | WDFW87                       | NWIFC87 | WDFW95 | WDFW95 | Combined |
| Fork Length<br>(FL)  | n         | 1,309                        | 62      | 197    | 289    | 1,857    |
|                      | $\bar{x}$ | 58.4                         | 63.2    | 50.0   | 48.1   | 56.0     |
|                      | s         | 5.9                          | 3.0     | 8.1    | 9.9    | 8.1      |
|                      | CV        | 10.1%                        | 4.7%    | 16.3%  | 20.5%  | 14.5%    |
| Total Length<br>(TL) | $\bar{x}$ | 62.1                         | 66.3    | 53.5   | 51.1   | 59.6     |
|                      | s         | 6.2                          | 3.0     | 8.6    | 10.3   | 8.5      |
|                      | CV        | 9.9%                         | 4.6%    | 16.1%  | 20.1%  | 14.3%    |
| DIFF                 | $\bar{x}$ | 3.69                         | 3.11    | 3.56   | 3.01   | 3.55     |
|                      | s         | 1.01                         | 0.74    | 0.88   | 0.92   | 1.01     |
|                      | CV        | 27.3%                        | 23.8%   | 24.6%  | 30.5%  | 28.3%    |
| FL, TL               | r         | 0.987                        | 0.970   | 0.996  | 0.997  | 0.994    |
| FL, DIFF             | r         | 0.195                        | 0.002   | 0.496  | 0.407  | 0.311    |

| Variable             | Statistic | Data Set: 68 cm ≤ FL ≤ 80 cm |         |        |        |          |
|----------------------|-----------|------------------------------|---------|--------|--------|----------|
|                      |           | WDFW87                       | NWIFC87 | WDFW95 | WDFW95 | Combined |
| Fork Length<br>(FL)  | n         | 1,109                        | 29      | 1      | 27     | 1,166    |
|                      | $\bar{x}$ | 74.1                         | 72.2    | 70     | 72.7   | 74.0     |
|                      | s         | 3.7                          | 3.0     |        | 3.9    | 3.7      |
|                      | CV        | 5.0%                         | 4.2%    |        | 5.4%   | 5.0%     |
| Total Length<br>(TL) | $\bar{x}$ | 78.0                         | 74.8    | 75     | 76.0   | 77.9     |
|                      | s         | 3.8                          | 3.1     |        | 3.8    | 3.8      |
|                      | CV        | 4.9%                         | 4.2%    |        | 5.1%   | 4.9%     |
| DIFF                 | $\bar{x}$ | 3.88                         | 2.65    | 5      | 3.26   | 3.84     |
|                      | s         | 1.04                         | 0.63    |        | 1.32   | 1.06     |
|                      | CV        | 26.9%                        | 23.7%   |        | 40.4%  | 27.7%    |
| FL, TL               | r         | 0.962                        | 0.980   |        | 0.943  | 0.961    |
| FL, DIFF             | r         | -0.045                       | 0.081   |        | -0.243 | -0.030   |



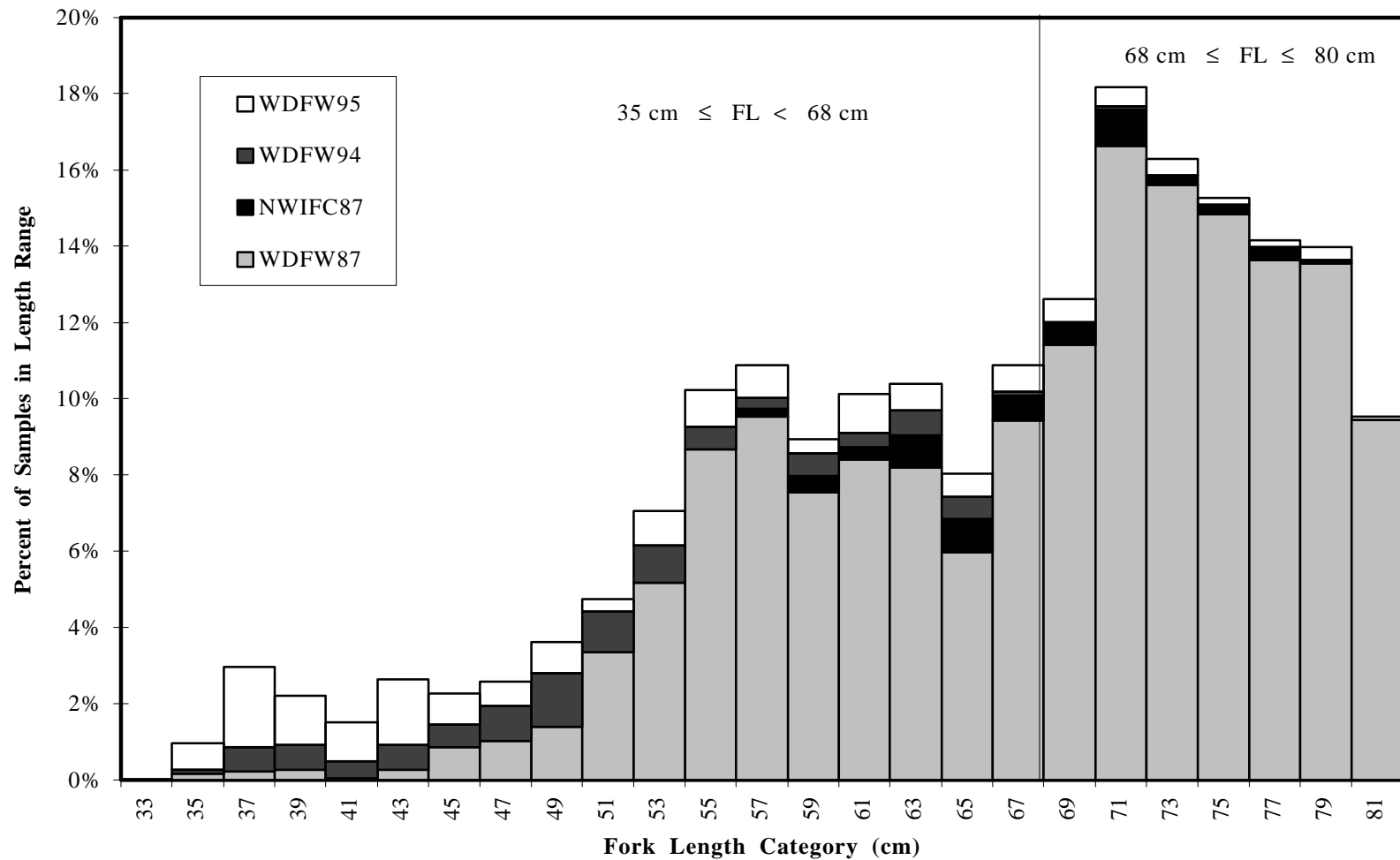


Figure 6. Frequency histogram for FL data used to estimate the relationship between FL and TL. Distributions represent the percent of samples in each length range:  $35 \text{ cm} \leq \text{FL} < 68 \text{ cm}$  and  $68 \text{ cm} \leq \text{FL} \leq 80 \text{ cm}$ . Contribution of each data set shown, also.

The slope estimates for the GMR model ( $\hat{b}_{GMR}$ ) and 95% confidence interval for each estimated slope are shown in Figure 7 for each data set (length range:  $35 \text{ cm} \leq \text{FL} \leq 80 \text{ cm}$ ). These results support the ANCOVA because the confidence interval around  $\hat{b}_{GMR}$  for each data set does not include the point estimates for the slope of the other data sets (Table 4). This supports the decision not to combine the four data sets across the entire  $35 \text{ cm} \leq \text{FL} \leq 80 \text{ cm}$  length range.

The results of the GMR analysis of the separate length ranges are summarized in Figure 8 and Table 4. For the different data sets, there is considerable overlap among the 95% confidence intervals for  $\hat{b}_{GMR}$ . The dotted lines in Figure 8 represent the  $\hat{b}_{GMR}$  estimate for the data sets combined over the indicated length range. These values are encompassed by the 95% confidence interval of each of the data sets.

The final estimation equations<sup>3</sup> for converting FL to TL are, for  $35 \text{ cm} \leq \text{FL} < 68 \text{ cm}$ :

$$TL = 1.023 + (1.045 FL) \quad [3]$$

and for  $68 \text{ cm} \leq \text{FL} \leq 80 \text{ cm}$ :

$$TL = 1.488 + (1.032 FL). \quad [4]$$

Because the GMR model is symmetric in  $X$  and  $Y$ , the TL to FL parameters can be estimated from the above equations by solving for FL. If

$$TL = \hat{a} + (\hat{b} FL)$$

then

$$FL = \left( \frac{TL}{\hat{b}} \right) - \left( \frac{\hat{a}}{\hat{b}} \right)$$

when converting from TL to FL. Therefore, the GMR equations for converting from TL to FL are, for  $37.6 \text{ cm} \leq \text{TL} < 71.7 \text{ cm}$ :

$$FL = (0.957 TL) - 0.979$$

and for  $71.7 \text{ cm} \leq \text{TL} \leq 84 \text{ cm}$ :

$$FL = (0.969 TL) - 1.442 .$$

---

<sup>3</sup> Parameter values for these conversion equations are reported to three significant digits. There is essentially no change in predicted values for FL or TL if four or five significant digits are used (all differences are within  $\pm 0.02 \text{ cm}$ ). If two significant digits are used predicted lengths often change by 0.10 to 0.30 cm.

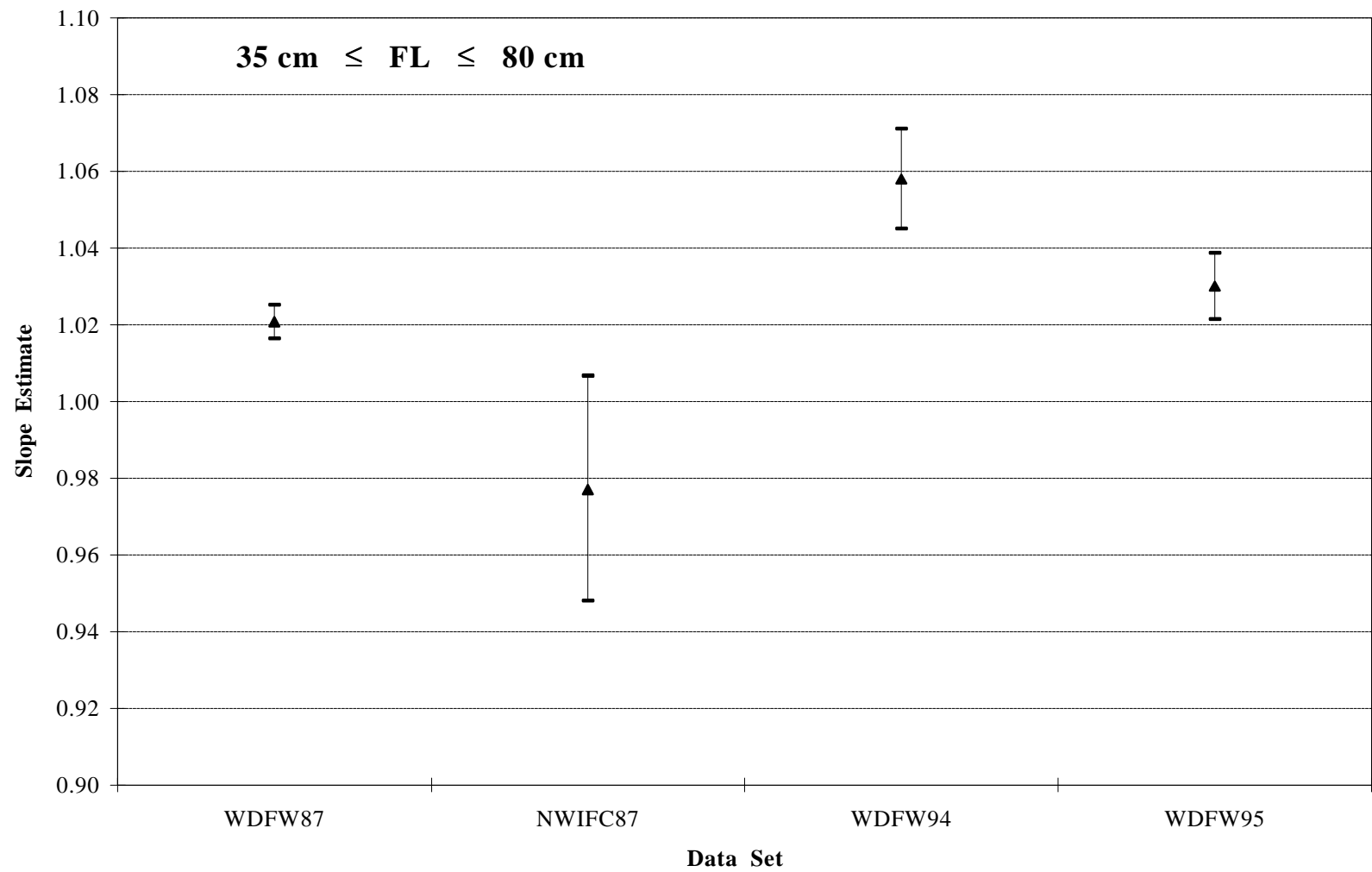


Figure 7. Estimated slope and 95% confidence interval for the GMR model for each data set. Length range:  $35 \text{ cm} \leq \text{FL} \leq 80 \text{ cm}$ .

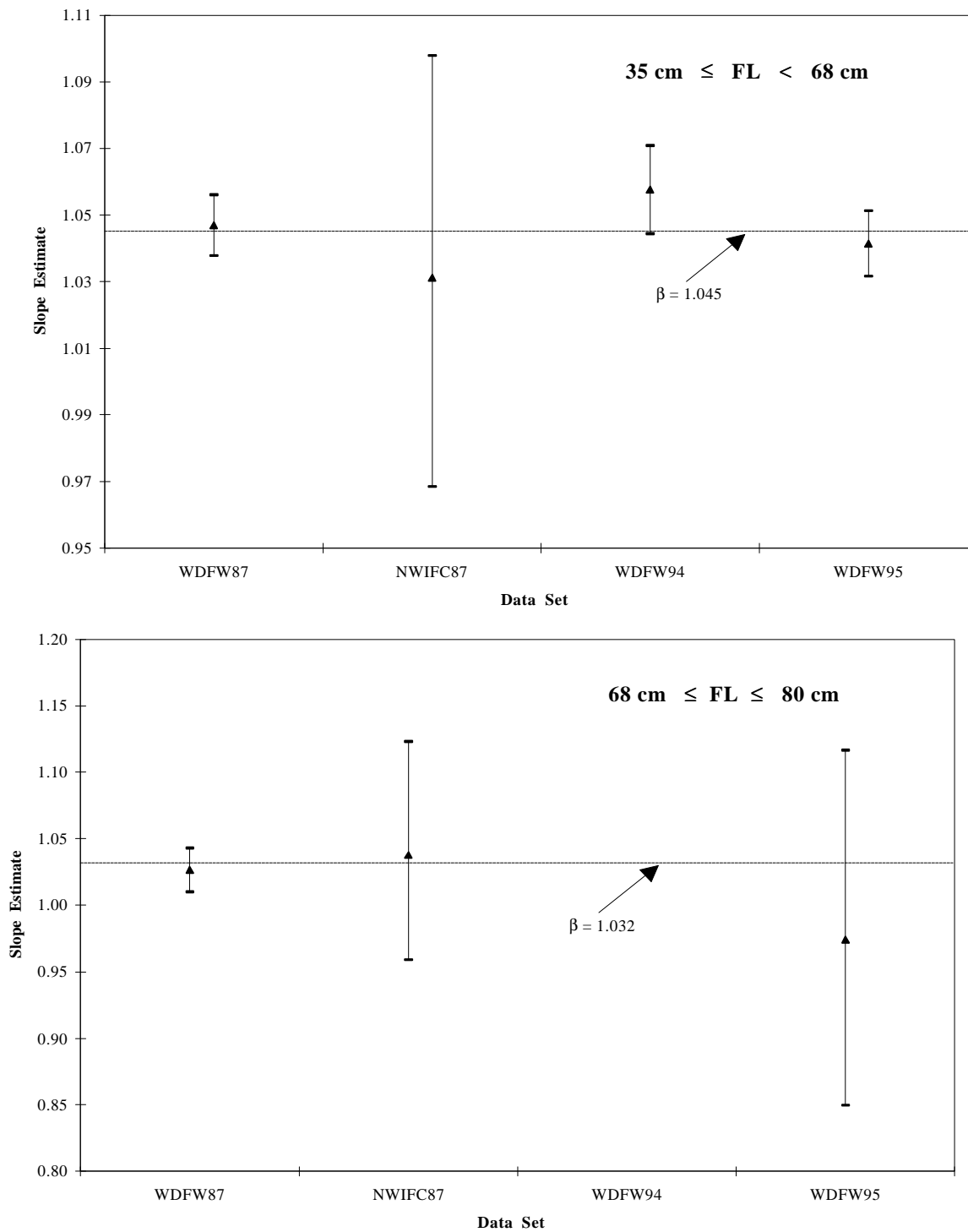


Figure 8. Estimated slope and 95% confidence interval for the GMR model for each data set within the two length ranges. There was only one data pair with  $FL \geq 68$  cm for the WDFW94 data set so no analysis was conducted.

Table 4. GMR model parameter estimates and 95% confidence interval for  $\hat{b}_{GMR}$  for each length range and data set analyzed.

| Data Set             | Length Range  | $\hat{b}$    | 95% Con. Int.        | $\hat{a}$    |
|----------------------|---|--------------|----------------------|--------------|
| WDFW87               | 35 cm $\leq$ FL $\leq$ 80 cm                              | 1.021        | 1.017 - 1.025        | 2.408        |
| NWIFC87              | "   | 0.977        | 0.948 - 1.007        | 4.484        |
| WDFW94               | "   | 1.058        | 1.045 - 1.071        | 0.660        |
| WDFW95               | "   | 1.030        | 1.022 - 1.039        | 1.524        |
| WDFW87               | 35 cm $\leq$ FL < 68 cm                                   | 1.047        | 1.038 - 1.056        | 0.948        |
| NWIFC87              | "   | 1.031        | 0.969 - 1.098        | 1.135        |
| WDFW94               | "   | 1.058        | 1.044 - 1.071        | 0.680        |
| WDFW95               | "   | 1.041        | 1.032 - 1.051        | 1.020        |
| <b>Data Combined</b> | <b>35 cm <math>\leq</math> FL &lt; 68 cm</b>              | <b>1.045</b> | <b>1.040 - 1.050</b> | <b>1.023</b> |
| WDFW87               | 68 cm $\leq$ FL $\leq$ 80 cm                              | 1.026        | 1.010 - 1.043        | 1.923        |
| NWIFC87              | "   | 1.038        | 0.959 - 1.123        | -0.089       |
| WDFW94               | " <sup>a</sup>  |              |                      |              |
| WDFW95               | "   | 0.974        | 0.850 - 1.117        | 5.136        |
| <b>Data Combined</b> | <b>68 cm <math>\leq</math> FL <math>\leq</math> 80 cm</b> | <b>1.032</b> | <b>1.015 - 1.048</b> | <b>1.488</b> |

<sup>a</sup> Only one data point so analysis not conducted.

The estimates for the slope ( $\hat{b}_{GMR}$ ) and intercept ( $\hat{a}_{GMR}$ ) for all these conversion equations are significantly different from zero ( $P \leq 0.05$ ). Although omitting the intercept would simplify the models, we do not feel this is appropriate given the significance of the tests of the hypothesis that the intercept equals zero.

Appendix Table 1 provides a summary of FL-to-TL conversions using equations 3 and 4 and compares these new estimates to estimates from the previous PMFC model. As expected, the PMFC model estimates of TL are larger for each FL value (Figure 9). The average difference between the models is +1.25 cm (range 1.08 cm to 1.43 cm) for length range I ( $35 \text{ cm} \leq \text{FL} < 68 \text{ cm}$ ) and +2.03 cm (range 1.08 cm to 2.17 cm) for length range II ( $68 \text{ cm} \leq \text{FL} \leq 80 \text{ cm}$ ) with the PMFC model always predicting the larger TL.

Although the estimated slopes ( $\hat{b}_{GMR}$ ) for the two FL-to-TL conversion equations are not significantly different ( $t$ -test,  $P > 0.05$ ), we recommend that separate conversion equations be used for each fork length range. The ANCOVA and the comparison of the slopes for the individual data sets indicate that the sets should not be combined across the entire  $35 \text{ cm} \leq \text{FL} \leq 80 \text{ cm}$  length range. Table 5 compares the predicted total length for larger FL values using the conversion equation for length range I to the values predicted for the conversion equation for length range II. The TL values predicted from the equation specific for larger fork lengths (equation 4) are about 0.5 cm smaller than the TL values predicted by the equation for the smaller fork lengths (equation 3).

Table 5. Comparison of predicted total lengths from the GMR FL-to-TL equations for length ranges I ( $35 \text{ cm} \leq \text{FL} < 68 \text{ cm}$ ) and II ( $68 \text{ cm} \leq \text{FL} \leq 80 \text{ cm}$ ).

| Fork Length (cm) | Predicted TL (cm)<br>Equation 3 | Predicted TL (cm)<br>Equation 4 |
|------------------|---------------------------------|---------------------------------|
| 70               | 74.17                           | 73.73                           |
| 75               | 79.40                           | 78.89                           |
| 80               | 84.62                           | 84.05                           |

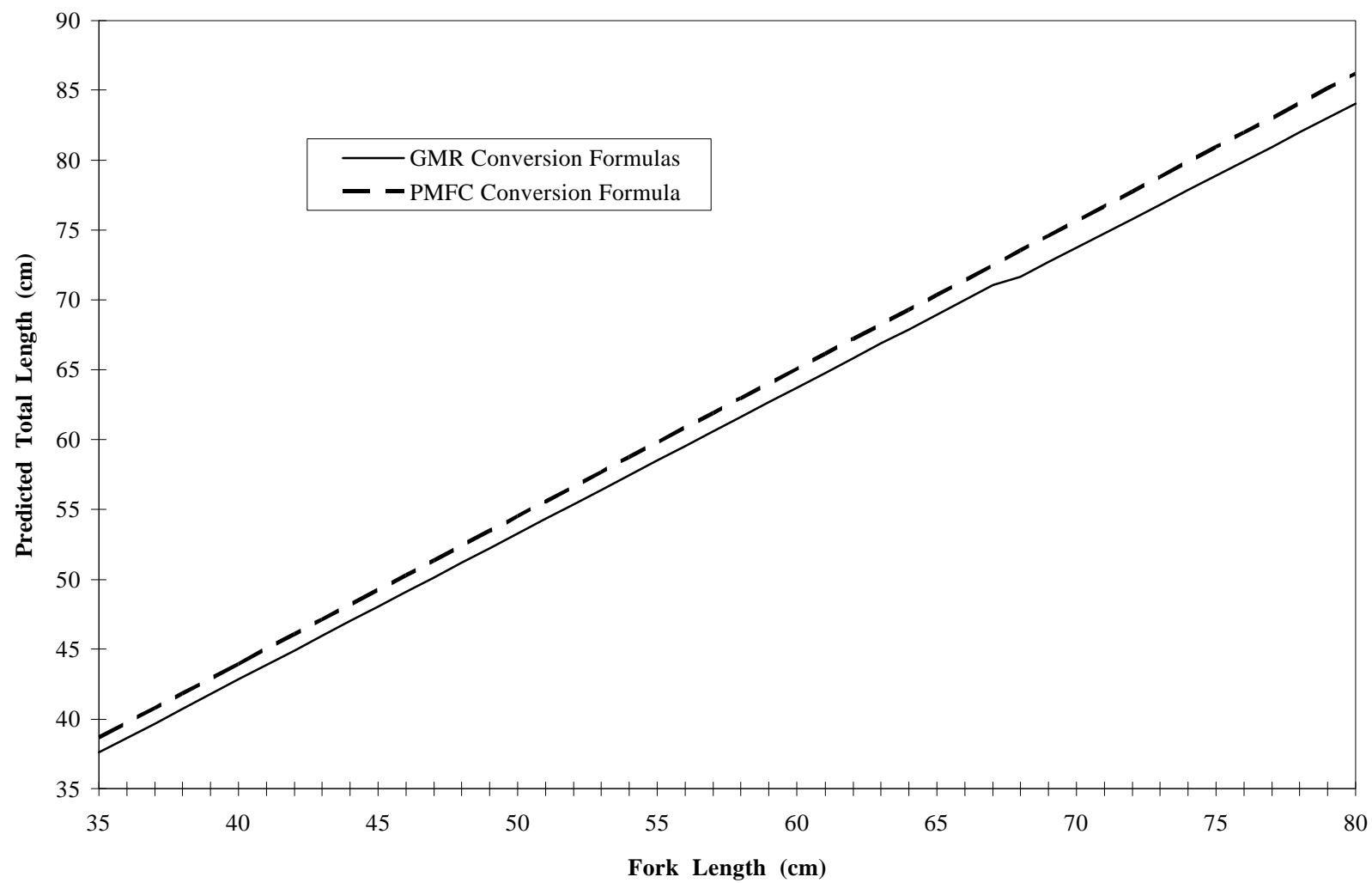


Figure 9. Comparison of GMR fork length to total length conversion lines to the line from the PMFC model.

## **DISCUSSION**

The recommended conversion equations will result in changes to the parameters used in fishery regulation assessment models. For example, the 1993 Treaty Troll Area 3/4/4B chinook fishery required a minimum total length of 22 inches. This is currently represented as a 51.3 cm fork length based upon the PMFC model. The new conversion equations result in a 52.5 cm fork length. Changes in model parameters, similar to these, should improve the models' ability to accurately project the actual impacts of proposed fishery regulations.

In addition, the effectiveness of the current size limits in some fisheries may need to be reassessed if the size limit was based upon the PMFC fork length to total length conversion. Minimum size limits may need to be lowered to achieve the desired management objectives.

The new conversions presented in this report will provide a consistent means to convert between fork length and total length for fishery managers in Washington. The conversions are based on data collected using consistent measuring techniques. The estimation method employed provides symmetric conversion formulas that take into account errors in both measurements.



## **ACKNOWLEDGMENTS**

The authors thank Kit Rawson, harvest management biologist for the Tulalip Tribe; Marianna Alexandersdottir, biometrician for the Washington Department of Fish and Wildlife; and Nick Lampsakis, senior fisheries management biologist for the Point No Point Treaty Council; for their timely and helpful reviews of earlier drafts of this report.

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Appendix Table 1. Comparison of fork length (FL) to total length (TL) conversions between the GMR model and the PMFC model.

| FL (cm) | GMR Model:<br>TL (cm) | PMFC Model:<br>TL (cm) |
|---------|-----------------------|------------------------|
| 35      | 37.60                 | 38.69                  |
| 36      | 38.64                 | 39.74                  |
| 37      | 39.69                 | 40.80                  |
| 38      | 40.73                 | 41.85                  |
| 39      | 41.78                 | 42.91                  |
| 40      | 42.82                 | 43.97                  |
| 41      | 43.87                 | 45.02                  |
| 42      | 44.91                 | 46.08                  |
| 43      | 45.96                 | 47.14                  |
| 44      | 47.00                 | 48.19                  |
| 45      | 48.05                 | 49.25                  |
| 46      | 49.09                 | 50.30                  |
| 47      | 50.14                 | 51.36                  |
| 48      | 51.18                 | 52.42                  |
| 49      | 52.23                 | 53.47                  |
| 50      | 53.27                 | 54.53                  |
| 51      | 54.32                 | 55.59                  |
| 52      | 55.36                 | 56.64                  |
| 53      | 56.41                 | 57.70                  |
| 54      | 57.45                 | 58.75                  |
| 55      | 58.50                 | 59.81                  |
| 56      | 59.54                 | 60.87                  |
| 57      | 60.59                 | 61.92                  |
| 58      | 61.63                 | 62.98                  |
| 59      | 62.68                 | 64.04                  |
| 60      | 63.72                 | 65.09                  |
| 61      | 64.77                 | 66.15                  |
| 62      | 65.81                 | 67.20                  |
| 63      | 66.86                 | 68.26                  |
| 64      | 67.90                 | 69.32                  |
| 65      | 68.95                 | 70.37                  |
| 66      | 69.99                 | 71.43                  |
| 67      | 71.04                 | 72.49                  |
| 68      | 71.66                 | 73.54                  |
| 69      | 72.70                 | 74.60                  |
| 70      | 73.73                 | 75.65                  |
| 71      | 74.76                 | 76.71                  |
| 72      | 75.79                 | 77.77                  |
| 73      | 76.82                 | 78.82                  |
| 74      | 77.86                 | 79.88                  |
| 75      | 78.89                 | 80.94                  |
| 76      | 79.92                 | 81.99                  |
| 77      | 80.95                 | 83.05                  |
| 78      | 81.98                 | 84.10                  |
| 79      | 83.02                 | 85.16                  |
| 80      | 84.05                 | 86.22                  |



